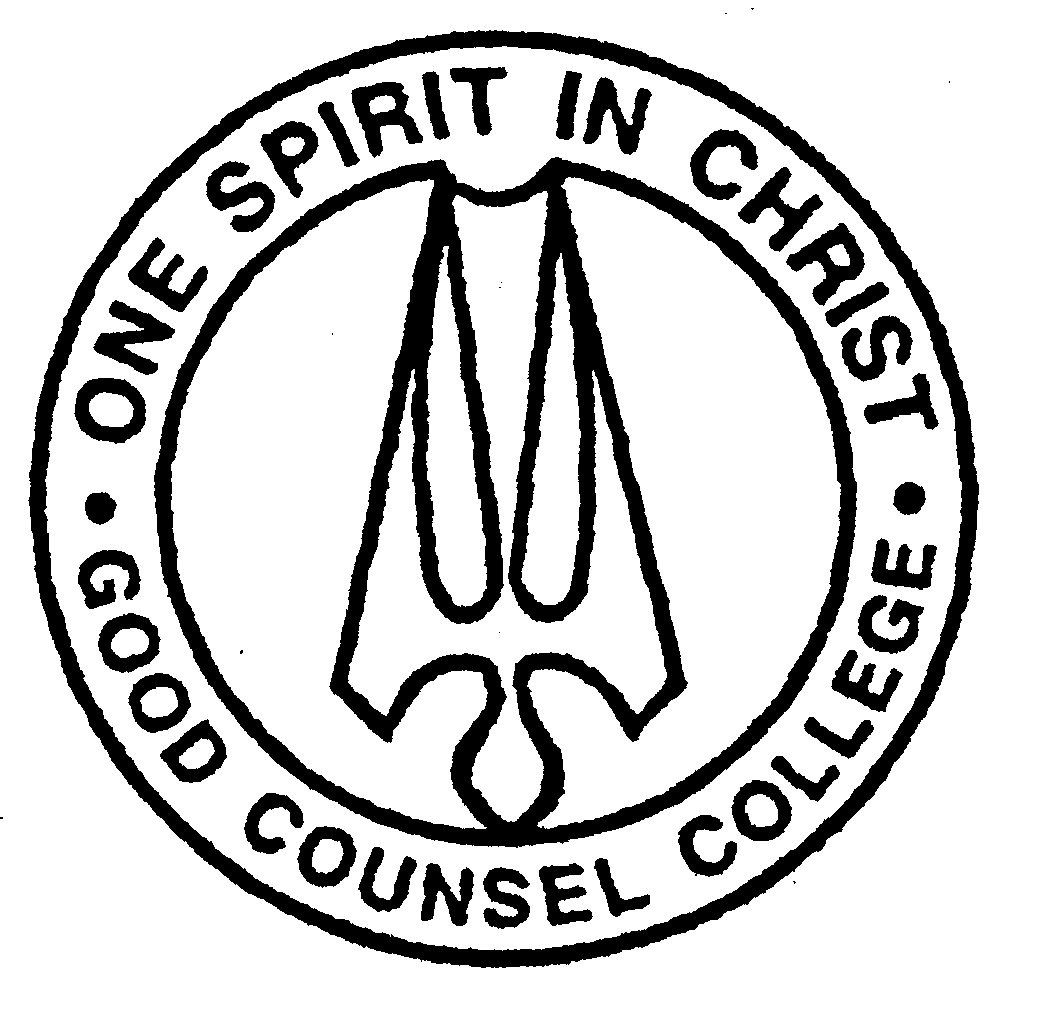
#### GOOD COUNSEL COLLEGE



#### YEAR 10 ADVANCED MATHS

#### PROBLEM-SOLVING AND MODELLING TASK

#### TERM 3

**NAME:** James Macgillivray

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| --- | --- | --- | --- |
| **Subject** | Advanced Maths | **Instrument no.** | Summative internal assessment 1 |
| **Technique** | Problem-solving and modelling task | | |
| **Unit** | Unit 10.3 | | |
| **Topic** | Topic: Quadratic Equations | | |

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| --- | --- | --- | --- | --- | --- | --- | --- |
| **Conditions** | | | | | | |  |
| **Duration** | 3 weeks (including 3 lessons of class time) | | | | | |  |
| **Mode** | Written report | | **Length** | | Up 5 pages in size 11 font (excluding appendixes) | | |
| **Individual/**  **group** | Individual | | **Other** | | - | |  |
| **Resources**  **available** | The use of technology is required, e.g.   * spreadsheet program and graphing software * graphics calculator * internet access | | | | | |  |
| **Context** | | | | | | |  |
| Many problems (not just mathematical ones) have multiple solutions but often the solution required is the one that is in some sense the ‘best’ and also satisfies certain restrictions. For example, if the problem is to devise a three-course dinner that would ‘satisfy’ all members of your family on a budget of $50, then the constraint is the budget and the ‘best’ solution is one that satisfies the tastes of all family members. There may be multiple solutions in this case or there may be none at all! | | | | | | |  |
| **Task** | | | | | | |  |
| Consider the following mathematical problem:  ***A rectangular animal enclosure with* n *parallel pens is to be made from a length of fencing* L*. Determine the dimensions of the pen so that the area is* (i) *maximised and* (ii) *the domain of reasonable solutions.***  You are to investigate the problem both algebraically and numerically. You will need to make and clearly state all assumptions used in solving the problem. You will need to determine the equation for the total area of your pen and find the dimensions that maximise and minimise the total area using spreadsheet and tech calculations. Your teacher will assign you particular values for *n* and *L*.  **10Advanced** You must use a variety of techniques to Evaluate and Verify your answers.  My values are n=6 L=85  My pen looks like = | | | | | | |  |
| **To complete this task, you must** | | | | | | |  |
| * present your findings as an investigative report based on the approach to problem-solving and mathematical modelling outlined the problem-solving flow chart (attached). * develop a unique response to the problem. | | | | | | |  |
| **Stimulus** | | | | | | |  |
|  | | | | | | |  |
| **Checkpoints** | | | | | | |  |
| 1 Week after issue  2 Weeks after issue  3 Weeks after issue (assessment submission) | | | | | | |  |
| **Criterion** | | | | **Marks allocated** | | **Result** |  |
| **Formulate**  Assessment objectives 1, 2, 5 | | | | **4** | |  |  |
| **Solve**  Assessment objectives 1, 6 | | | | **7** | |  |  |
| **Evaluate and verify**  Assessment objectives 4, 5 | | | | **5** | |  |  |
| **Communicate**  Assessment objectives 3 | | | | **4** | |  |  |
| **Total** | | | | **20** | |  |  |
| **Authentication strategies** | | * Student progress will be documented and copies of student responses collected at the checkpoints. * Each student will produce a unique response by using individualised data and producing unique reports. * Students must submit a declaration of authenticity (see below). | | | | |  |
| **Scaffolding** | | The approach to problem-solving and modelling on the following page must be used. | | | | |  |

**Declaration of authenticity:**

I declare that is work is my own and that help, including the extent of this help, received from other persons has been fully described and acknowledged in the report.

Signed: James Macgillivray Date: 2/9/2022

**Approach to Problem-Solving & Mathematical Modelling**

|  |  |
| --- | --- |
|  | Once you understand what the problem is asking, you must design a plan to solve the problem. Translate the problem into a mathematically purposeful representation by first determining the applicable mathematical and/or statistical principles, concepts, techniques and technology that are required to make progress with the problem, then list the mathematical techniques and procedures you will use to develop the response. Consider how you will determine the data values that lie on your chosen curve and what methods you will use to generate the model. Appropriate assumptions, variables and observations must be identified and documented, based on the logic of a proposed solution and/or model.  In mathematical modelling, formulating a model involves the process of mathematisation ─ moving from the real world to the mathematical world. |
| Select and apply mathematical and/or statistical procedures, concepts and techniques previously learnt to solve the mathematical problem to be addressed through your model. Possible approaches are wide-ranging and include synthesising and refining the polynomial model and generating and testing the feasibility of the polynomials of higher order, as well as using standard mathematical techniques.  This process may require returning to your initial observations and assumptions and reconsidering and modifying them to ensure the problem has been solved or can actually be solved.  Solutions can be found using algebraic, graphic and technological methods. |
| Icon  Description automatically generatedOnce a possible solution has been achieved, you need to consider the reasonableness of the solution and/or the utility of the model in terms of the problem. Evaluate your results and make a judgment about the solution(s) to the problem in relation to the original issue, statement or question.  This involves exploring the strengths and limitations of your solution and/or model. Where necessary, this will require going back through the process to further refine your solution and/or model. In mathematical modelling, you must check that the output of your model provides a valid solution to the real-world problem it has been designed to address.  Use both a residual analysis and the correlation coefficient to interpret the results of the mathematics compared with the original task. |
| Icon  Description automatically generated with low confidenceThe development of solutions and models to abstract and real-world problems must be capable of being evaluated and used by others and so need to be communicated clearly and fully. Communicate your findings systematically and concisely using mathematical, statistical and everyday language in a structured report. Draw conclusions, discussing the key results and the strengths and limitations of the solution and/or model. You could offer further explanation, justification, and/or recommendations, framed in the context of the initial problem. |
|  |

**Instrument-specific marking guide (ISMG)**

|  |  |  |
| --- | --- | --- |
| **Criterion: Formulate (F) The student work has the following characteristics:** | **Marks** |  |
| • documentation of appropriate assumptions  • accurate documentation of relevant observations  • accurate translation of all aspects of the problem by identifying mathematical concepts and techniques. | 3–4 |  |
| • statement of some relevant observations  • translation of simple aspects of the problem by identifying mathematical concepts and techniques. | 1–2 |  |
| • does not satisfy any of the descriptors above. | 0 |  |
| **Criterion: Solve (S) The student work has the following characteristics:** | **Marks** |  |
| • use of complex procedures to reach an accurate solution  • discerning application of mathematical concepts and techniques relevant to the task  • efficient use of technology. | 6–7 |  |
| • use of complex procedures to reach a reasonable solution  • application of mathematical concepts and techniques relevant to the task  • appropriate use of technology. | 4–5 |  |
| • use of simple procedures to make some progress towards a solution  • simplistic application of mathematical concepts and techniques relevant to the task  • superficial use of technology. | 2–3 |  |
| • isolated use of technology or procedures relevant to the task. | 1 |  |
| • does not satisfy any of the descriptors above. | 0 |  |
| **Criterion: Evaluate and verify (EV) The student work has the following characteristics:** | **Marks** |  |
| • justification and explanation of procedures used and decisions made using mathematical reasoning  • evaluate the reasonableness of solutions by considering the results, assumptions and/or observations  • evaluation of relevant strengths and limitations of the solution and/or model. | 4–5 |  |
| • explanation of procedures used and decisions made  • statement about the reasonableness of solutions by considering the context of the task  • statement about relevant strengths and limitations of the solution and/or model. | 2–3 |  |
| • statement about procedures, decisions and/or the reasonableness of solutions. | 1 |  |
| • does not satisfy any of the descriptors above. | 0 |  |
| **Criterion: Communicate (C) The student work has the following characteristics:** | **Marks** | |
| • correct use of appropriate mathematical, statistical and everyday language and conventions to develop the response  • coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion, which can be read independently of the task sheet. | 3–4 | |
| • use of some appropriate mathematical, statistical and everyday language and conventions to develop the response  • adequate organisation of the response. | 1–2 | |
| • does not satisfy any of the descriptors above. | 0 | |

|  |  |  |
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| **Criterion** | **Marks allocated** | **Result** |
| **Formulate** Assessment objectives 1, 2, 5 | **4** |  |
| **Solve** Assessment objectives 1, 6 | **7** |  |
| **Evaluate and verify** Assessment objectives 4, 5 | **5** |  |
| **Communicate** Assessment objectives 3 | **4** |  |
| **Total** | **20** |  |

**Introduction**

One of the concepts in mathematics modelling is quadratic equations. Quadratics equations are a type of polynomial which when graphed creates a parabola shape and have a general formula of ax2+bx+c=0. These can be used in many maths problems especially involving area. The problem given is to find the maximum area for a fenced area with L amount of fence and n amount of pen within that fenced area with fencing separating each pen.

**Formulate (Trial and Error)**

Using the trial error method I chose for length and width to produce a range of where the answer could be. I *observed* that not all the or too much fence was used in the in the solution meaning the solution could not possibly be correct. To determine an answer I substituted the length and width into both the perimeter and area formula to check whether then dimensions fit the fence length constraint and find the total area. When formulating the trial and error I *assumed* that the diagram drawn of the question what the question was asking and that my formula of the length of fence is true.

**Solve**

|  |  |
| --- | --- |
| Variables defined in Task  Formula for finding length of fence used  Random numbers where chosen for length and width  A Solution with area and length of fence used |  |

**Evaluation**

The answer supplied by this formula clearly gives a non optimal answer for the given parameters as it does not use all of the fence which makes means that the at least some area can be added by adding a small amount of length or width. Strengths of the solve are the speed of the solve to get an approximate solution. Limitations are that finding the correct solution would take a too long for it to be a viable method.

**Formulate (Spreadsheet)**

Using the quadratic created in the previous section I imported that into a spreadsheet tool and charted it to create a graph. This was used to find the ideal width and area which was substituted to create an answer.

**Solve**

|  |  |
| --- | --- |
| Variables defined in task  Formula for fence length in terms of length and width  Rearrange to make length subject  Formula for area in terms of length and width  Substitute the fence length into the area formula  Rearrange to create quadratic  Graph of Quadratic in spreadsheet with  =A2^2\*(-3.5)+42.5\*A2 as formula  Turning point derived from spreadsheet  Substitute width and area to find length  length of ideal area |  |

**Evaluation**

The answer provided by this method was very close to the real answer but is slightly off by about two ten-thousandths. Strengths of this solve are its speed and its relatively high accuracy. Limitations are that finding the real answer would be almost impossible as the width is a number with infinite decimals.

**Formulate (Quadratics)**

Using substitution, I created a quadratic which when graphed displays the area as Y when the width is X. Using the turning point of this quadratic I found the maximum possible area given a certain width. This gave me two numbers of the area of a rectangle formula which I used to find the other length. In this solve I *assumed* that the turning point gave me the maximum area and that the fenced area was completely rectangular.

**Solve**

|  |  |
| --- | --- |
| Variables defined in task  Formula for fence length in terms of length and width  Rearrange to make length subject  Formula for area in terms of length and width  Substitute the fence length into the area formula  Rearrange to get a general form quadratic  Find the turning point as that is where the maximum answer is located on the parabola (see figure 1)  Turning point formula to find the x of the turning point  Width of maximum area  Substitute width back into quadratic to find area  Maximum area  Substitute Area and width to find length  Length of maximum area |  |
|  |  |

**Evaluation (Quadratics)**

The answer provided by this solve maximises the area of the pens and is the correct solution the question. It is one of a few ways to solve this kind of question and all should produce the same answer. *Strengths* of this solve are are accuracy, the ability to solve without technology and to ability to quickly find the area with x width. *Limitations* are the speed of solving as doing manual calculation can be take quite a long time.

**Formulate (Technology)**

To verify my quadratics solution I used an online graphics calculator to find my turning point.

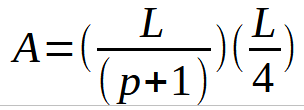
**Solve**

|  |  |
| --- | --- |
| Quadratic as derived from previous section  Quadratic as graphed in graphics calculator |  |

**Evaluation**

This gives an identical answer for the turning point to the quadratics thus verifying its results.

**Conclusion**

To find the maximum area of a two dimensional shape such as a rectangle quadratics are a useful tool to find the solution. A general formula was found from the solutions given from the quadratic which substantially reduces solve time. This formula is: